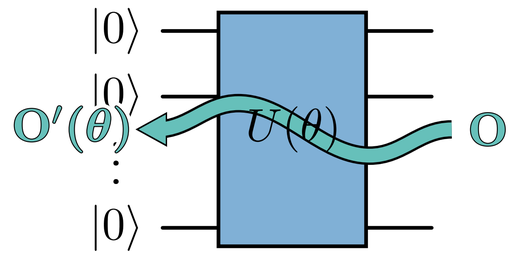
Training of Large Shallow Parametrized Quantum Circuits through Pauli Propagation

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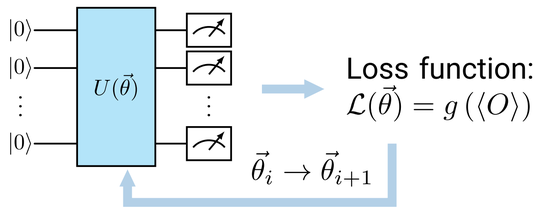
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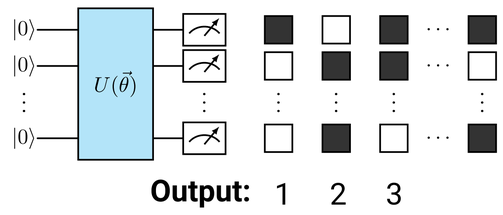
Use case

Obtain the **explicit** expectation value function of a circuit's observable in terms of its parameters. Useful when the loss is defined via expectation values but the output is a single measurement.

Training:



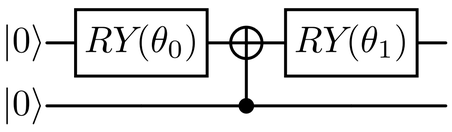
Output:



Propagation

Propagation of observables in the *Heisenberg picture*:

Circuit example:



Propagation rules:

$RY(\theta)$	$X \to \cos\theta X + \sin\theta Z$
	$Z \to \cos\theta Z - \sin\theta X$
⊕	$Z \otimes I \to Z \otimes Z$
	$X \otimes I \to X \otimes I$

Propagation of Z ⊗ I:

1 Action of $RY(\theta_1) \otimes I$

$$Z\otimes I \to \cos(\theta_1) Z\otimes I - \sin(\theta_1) X\otimes I$$

2 Action of CNOT$(1,0)$

$$\cos(\theta_1) Z\otimes I - \sin(\theta_1) X\otimes I \to \cos(\theta_1) Z\otimes Z - \sin(\theta_1) X\otimes I$$

3 Action of $RY(\theta_0) \otimes I$

$$\begin{aligned}\cos(\theta_1) Z\otimes Z - \sin(\theta_1) X\otimes I \to \\ \to + \cos(\theta_0)\cos(\theta_1) Z\otimes Z - \sin(\theta_0)\cos(\theta_1) X\otimes Z + \\ - \cos(\theta_0)\sin(\theta_1) X\otimes I - \sin(\theta_0)\sin(\theta_1) Z\otimes I\end{aligned}$$

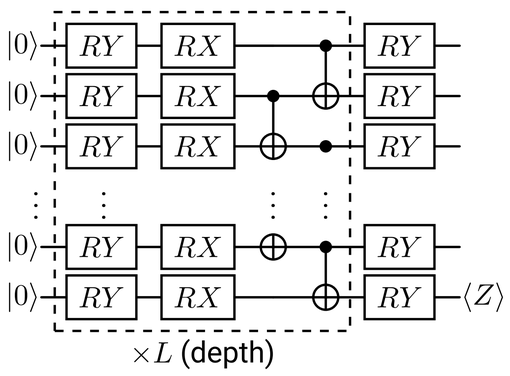
4 Trimming: Computing the expectation value of the Pauli Obs.

$$\langle Z\otimes I\rangle(\vec{\theta}) = +\cos(\theta_0)\cos(\theta_1) - \sin(\theta_0)\sin(\theta_1)$$

Exact Propagation

Number of qubits: 16

Studied circuit

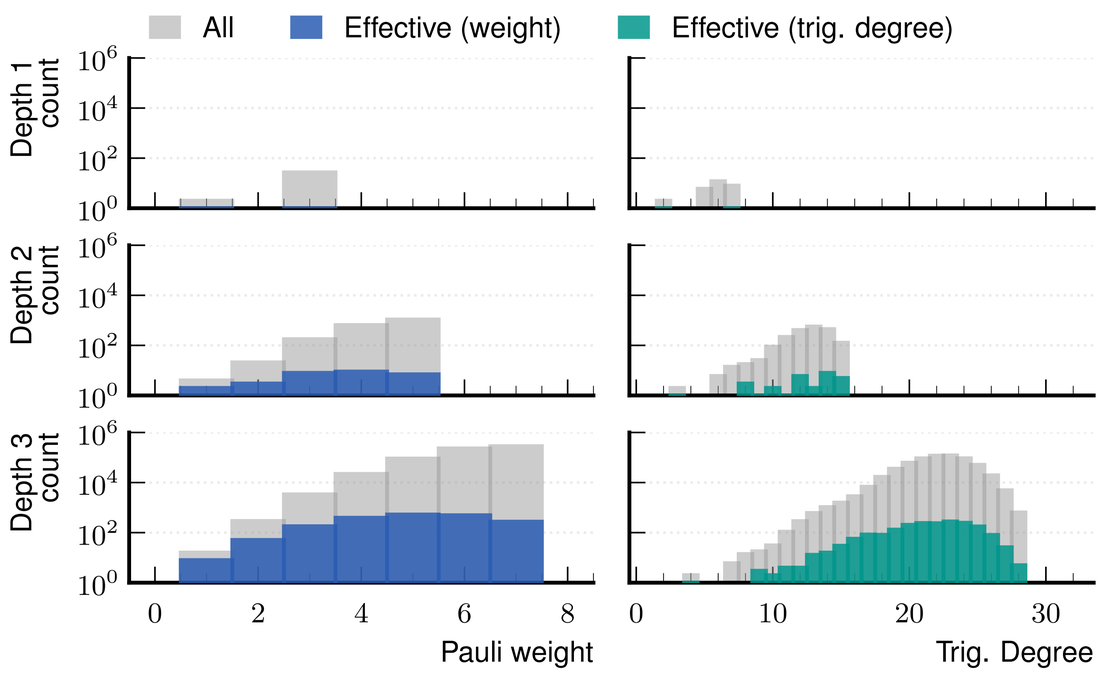


Quantities

- **Pauli weight**: number of non-identity Pauli-Op
- **Trig. Degree**: number of trigonometric functions multiplied

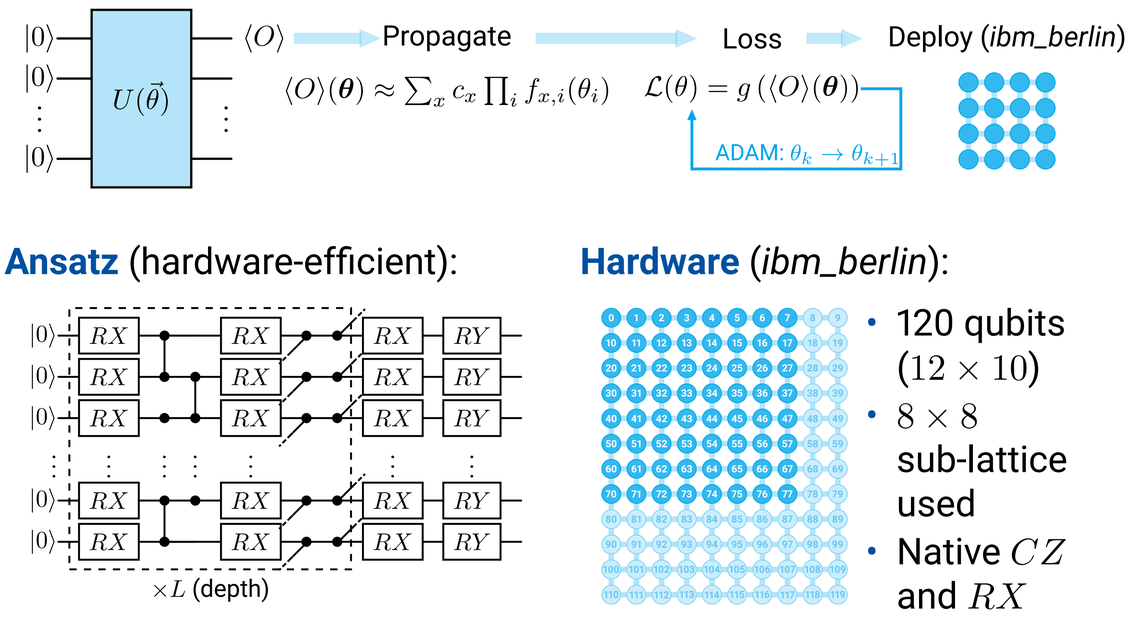
Example:

$$\underbrace{\sin\theta_8 \sin\theta_1 \cos\theta_0}_{\text{Trig. Degree } (3)} \quad \underbrace{X_0 Z_1}_{\text{Weight } (2)}$$



★ The number of terms grows rapidly → necessity of cutoffs: cut-off on Pauli weight w_{cut} and trigonometric degree δ_{cut}

Train & Deployment



Ansatz (hardware-efficient):

Hardware (*ibm_berlin*):

- 120 qubits (12×10)
- 8×8 sub-lattice used
- Native CZ and RX

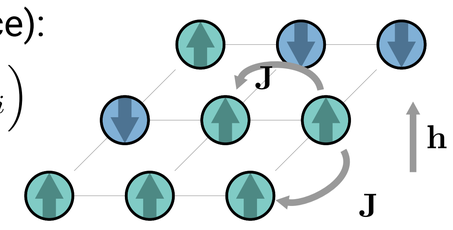
Application: VQE

Number of qubits: 64

Goal: Find the parameters $\vec{\theta}$ for an ansatz U such that $|\psi(\vec{\theta})\rangle = U(\vec{\theta})|0\rangle$ is as close as possible to the ground state of H.

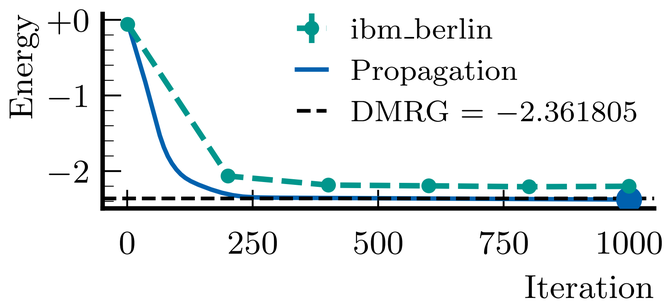
Hamiltonian (2D Ising, 8×8 square lattice):

$$H(J,h) = \frac{1}{N}\Big(- J \sum_{\langle i,j \rangle} Z_i Z_j - h \sum_i X_i \Big)$$



Loss (energy):

$$\mathcal{L}(\vec{\theta}) = E(\vec{\theta}) = \langle \psi(\vec{\theta}) | H | \psi(\vec{\theta}) \rangle$$



Setup:

- $w_{\text{cut}} = 12$
- $\delta_{\text{cut}} = -$
- Depth 2 (384 params)

★ Propagator gets very close to the DMRG ground state. Hardware run shows a small gap, likely dominated by noise.

Application: Generation

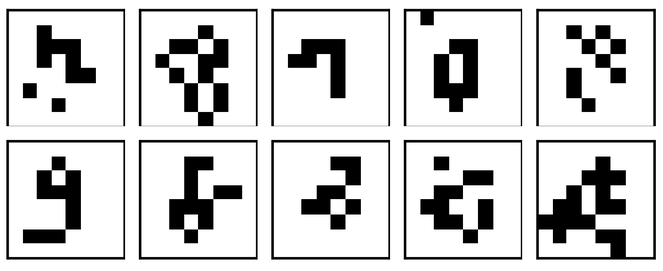
Number of qubits: 64

Goal: Train a QCBM: the measurement outcomes of should follow the distribution of binarized 8×8 (downscaled) MNIST images.

Loss (MMD^2 estimator, Gaussian kernel of bandwidth σ):

$$\mathcal{L}_{\text{MMD}}(\vec{\theta}) = \frac{1}{|\mathcal{A}|} \sum_j \Big(\langle Z_{a_j} \rangle_{\text{data}} - \langle Z_{a_j}(\vec{\theta}) \rangle \Big)^2, \quad \mathcal{A} = \{a_j \sim \mathcal{P}_\sigma\}$$

Samples (*ibm_berlin*, 1 shot = 1 image):



Setup:

- $w_{\text{cut}} = 12$
- $\delta_{\text{cut}} = -$
- Depth 1 (256 params)
- $|\mathcal{A}| = 1000$
- $\sigma \approx 3.74$

★ MMD is hard in this framework: many observables must be propagated each round, forcing truncation.

References

[1] *Symbolic Pauli Propagation for Gradient-Enabled Pre-Training of Quantum Circuits*, **arXiv:2512.16674**

[2] *Classical Pre-Training of Large Shallow Parametrized Quantum Circuits through Pauli Propagation and deployment on Quantum Hardware*, **arXiv:XXXX.XXXXX**

GitHub

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